Day 2

Problem-Solving Task

I have quarters, dimes, and nickels in my piggy bank. If I take three coins out of my piggy bank, how much could the three coins be worth?

Discuss solutions as well as: (1) Process standards addressed, (2) content standards addressed, (3) Webb's level of task.

RICH MATHEMATICAL TASK SORT ACTIVITY

Each pair will need the following three handouts: (1) Task Analysis Guide, (2) Categorizing Mathematical Tasks, and (3) Packet of Tasks A–P.

Task Analysis Guide:

There are two kinds of lower-level demands and two kinds of higher-level demands. Instead of discussing these in-depth, you will gain an understanding of these demands by sorting mathematical tasks.

Tasks A–P:

Some of these are middle-school tasks, but the idea is for you to think about the content our students will encounter in subsequent grades and what level-of-demand they require.

ACTIVITY

(1) Work in assigned pairs to sort Tasks A–P into two stacks: low-level and high-level (using descriptions on Task Analysis Guide).

(2) Re-sort the low-level stack into the two categories ("memorization" or "procedures without connections"). Do the same for your high-level stack.

(3) Record your sorting on the "Categorizing" handout. Be prepared to justify your decisions using the bulleted items on the Task Analysis Guide.

Task Analysis Guide

Lower-Level Demands

Memorization

- involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory.
- cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated.
- have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.

Procedures Without Connections

- are algorithmic. Use of the procedure is either specifically called or its use is evident based on prior instruction, experience, or placement of the task.
- require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- have no connection to the concepts or meaning that underlie the procedure being used.
- are focused on producing correct answers rather than developing mathematical understanding.
- require no explanations or explanations that focuses solely on describing the procedure that was used.

Task Analysis Guide (Continued)

Higher-Level Demands

Procedures With Connections

- focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- usually are represented in multiple way (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics

- require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- demand self-monitoring or self-regulation of one's own cognitive processes.
- require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Categorizing Mathematical Tasks

Indicate whether each task is low or high level by placing an "X" in the appropriate column. Then identify the task according to the subcategories of low or high level demanding task.

TASK	LOW-LEVEL	HIGH-LEVEL
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		
M		
N		
O		
P		

TASK A

Manipulatives or Tools Available: Calculator

Treena won a 7-day scholarship worth $1000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp she must choose between a week of individual instruction at $60 per day or a week of group instruction at $40 per day. Treena's food and other expenses are fixed at $45 per day. If she does not plan to spend any money other than the scholarship, what are all choices of travel and instruction plans she could afford to make? Explain which option you think Treena should select and why.

Source: Kenney and Silver 1997, p. 108

TASK B

Manipulatives or Tools Available: Counters

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show *all* your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.

(1^{st} step)	(2^{nd} step)	(3^{rd} step)
		• • • •
	• • •	• • • •
• •	• • •	• • • •
2 dots	6 dots	12 dots

Marcy has to determine the number of dots in the 20^{th} step, but she does not want to draw all 20 pictures and then count the dots.

Explain how she should do this *and* give the answer that Marcy should get for the number of dots.

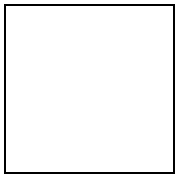
Source: Kenney and Silver 1997, p. 240.

TASK C

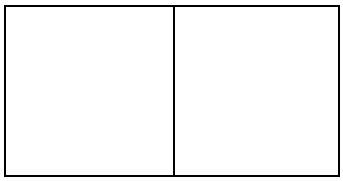
Manipulatives or Tools Available: Square Pattern Tiles

Using the side of a square pattern tile as a measure, find the perimeter (i.e., distance around) of each train in the pattern block figure shown below.

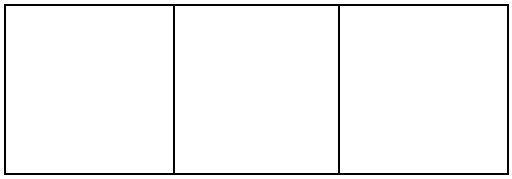
Train 1



Train 2



Train 3



TASK D

Manipulatives or Tools Available: None

Part A: The place kicker on the North High School football team has made 13 out of 20 field goals so far this season. The place kicker on the South High football team has made 15 out of 25 so far this season. Which player has made the greatest percentage of field goals?

Part B: If the "better" player does not play for the rest of the season, how many field goals would the other player need to make in order to take the lead in terms of greatest percentage of field goals?

TASK E

Manipulatives or Tools Available: Calculator

Divide using paper and pencil. Check your answer with a calculator and round the decimal to the nearest thousandth.

$$1.3\overline{)525}$$

$$7.25\overline{)52.75}$$

$$.12\overline{)30.459}$$

TASK F

Manipulatives or Tools Available: None

Match the property name with the appropriate equation.

1. Commutative property of addition	a. $r(s + t) = rs + rt$
2. Commutative property of multiplication	b. $x \cdot l/x = 1$
3. Associative property of addition	c. $–y + x = x + (-y)$
4. Associative property of multiplication	d. $a/b + -a/b = 0$
5. Identity property of addition	e. $y \cdot (zx) = (yz) \cdot x$
6. Identity property of multiplication	f. $l \cdot (xy) = (xy)$
7. Inverse property of addition	g. $d \cdot 0 = 0$ and $0 \cdot d = 0$
8. Inverse property of multiplication	h. $x + (b + c) = (x + b) + c$
9. Distributive property	i. $y + o = y$
10. Property of zero for multiplication	j. $p \cdot q = q \cdot p$

TASK G

Manipulatives or Tools Available: Base Ten Blocks, grid paper

.08 .8 .080 .008000

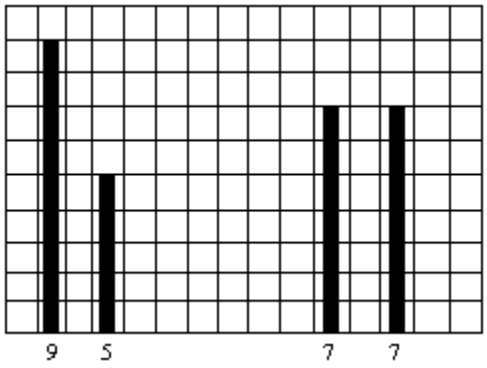
Make three observations about the relative size of the above 4 numbers. Be sure to explain your observations as clearly as possible. Feel free to illustrate your observations if you feel it would help others understand them.

Adapted from QUASAR Project – QUASAR Cognitive Assessment Instrument – Release task

TASK H

Manipulatives or Tools Available: Grid Paper

The pairs of numbers in a-d below represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.



a) 14 and 8 b) 16 and 7 c) 7 and 12 d) 13 and 15

By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total number of cubes is now distributed into 2 columns of equal height. And that is what the average means.

Taken from *Visual Mathematics Course I,* The Math Learning Center, 1995, Lesson 10, Follow-up Student Activity 10.1, #1, p. 121.

TASK I

Manipulatives or Tools Available: None

Write and solve a proportion for each.

17 is what percent of 68?

What is 15% of 60?

8 is 10% of what number?

24 is 25% of what number?

28 is what percent of 40?

What is 60% of 45?

36 is what percent of 90?

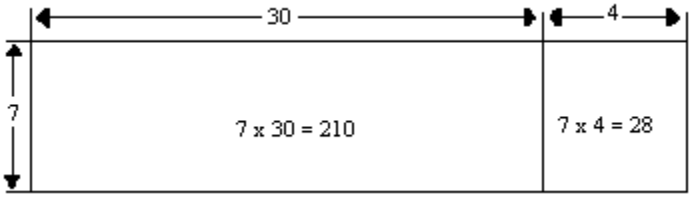
What is 80% of 120?

21 is 30% of what number?

TASK J

Manipulatives or Tools Available: None

One method of mentally computing 7 x 34 is illustrated in the diagram below:



Mentally compute these products. Then sketch a diagram that describes your methods for each.

a) 27 x 3

b) 325 x 4

Adapted from *Visual Mathematics Course I,* The Math Learning Center, 1995, Lesson 37, Focus Teacher Activity, pp. 457-63.

TASK K

Manipulatives or Tools Available: Calculator with scientific functions

Penny's mother told her that several of her great-great-great-grandparents fought in the Civil Way. Penny thought this was interesting and she wondered how many great-great-great grandparents that she actually had. When she found that number, she wondered how many generations back she'd have to go until she could count over 100 ancestral grandparents or 1000, or 10,000, or even 100,000. When she found out she was amazed and she was also pretty glad she had a calculator. How do you think Penny might have figured out all of this information? Explain and justify your method as clearly and completely as possible.

Adapted from Blackline Master 3B from *AW Mathematics: Mathematics with Calculators: Resources for Teachers, Grade 6*, Section 3, Activity 2. Addison-Wesley Publishing Company, 1988. Copyright 1988 Addison-Wesley Publishing Co., Inc.

TASK L

Manipulatives or Tools Available: Base-10 Blocks

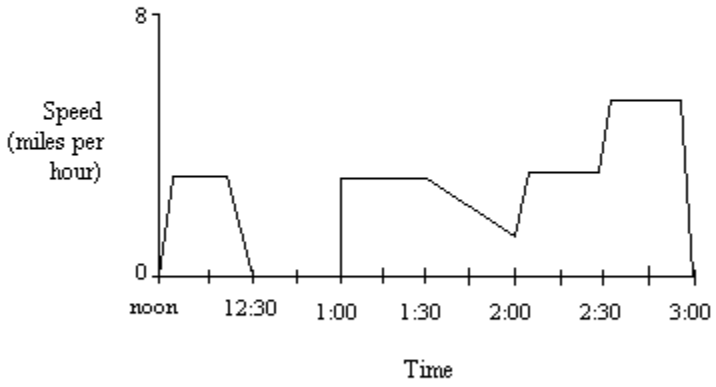
Using Base-10 Blocks, show that 0.292 is less than 0.3.

TASK M

Manipulatives or Tools Available: None

Use the following information and the graph to write a story about Tony's walk.

At noon Tony started walking to his grandmothers house. He arrived at her house at 3:00. The graph below shows Tony's speed in miles per hour throughout his walk.



Write a story about Tony's walk. In your story, describe what Tony might have been doing at the different times.

Taken from the QUASAR Project – QUASAR Cognitive Assessment Instrument – Release Task.

TASK N

Manipulatives/Tools: None

The cost of a sweater at J. C. Penney's was $45.00. At the "Day and Night Sale" it was marked down 30% off the original price. What was the price of the sweater during the sale? Explain the process used to find the sale price.

TASK O

Manipulatives or Tools Available: None

Give the fraction and percent for each decimal.

.20 = _____ = _____

.25 = _____ = _____

$.\overline{3}$ = _____ = _____

.50 = _____ = _____

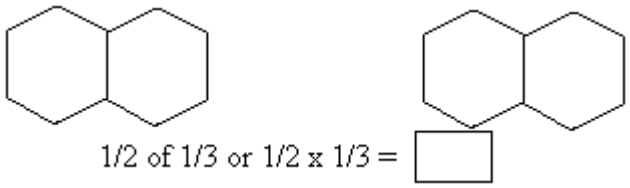
$.\overline{6}$ = _____ = _____

.75 = _____ = _____

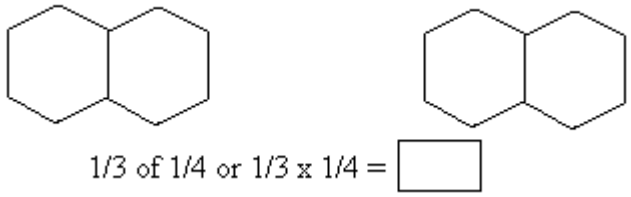
TASK P

Manipulatives or Tools Available: Pattern Blocks

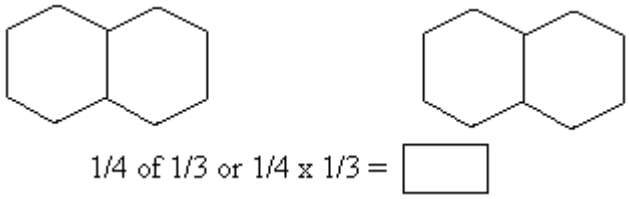
Find ½ of 1/3. Use pattern blocks. Draw your answer.



Find 1/3 of ¼. Use pattern blocks. Draw your answer.



Find ¼ of 1/3. Use pattern blocks. Draw your answer.



RICH MATHEMATICAL TASK SORT ACTIVITY

DEBRIEFING QUESTIONS

1. Based on your work with these tasks, what differentiates a low-level demanding task from a high-level demand task?

2. How do rich mathematical tasks fit with new TN standards?

Discussion Point: If we use higher-level demanding tasks, then our students will be able to do the lower-level demands because they are embedded in higher-level tasks; the converse is not true. Using only lower-level demand tasks does not mean our students can do higher-level demand tasks.

Developing Mathematical Thinking Through Questioning

What is an effective question?

•It is a question that probes for deeper meaning and often sets the stage for further questioning.

•It fosters the development of critical thinking skills and higher order capabilities.

•It promotes problem-solving and understanding.

•A good question is the principle component of designing inquiry-based learning.

Developing Mathematical Thinking Through Questioning

If you only ask for answers, you might get the correct response, but no assurance that the student understands the concept.

For example: What is 2^2 ?

The student says "4".

What if the student is thinking:
"Exponent multiplies base"
Answer is 4.

Of course, this student will also say that $3^2 = 6$!

Can you really answer questions correctly and have no clue what is happening?

QUIZ

The Monotillation of Traxoline

It is very important that you learn about traxoline. Traxoline is a new form of zionter. It is monotilled in Ceristanna. The Ceristannians gristerlate large amounts of fevon and then bracter it to quasel traxoline. Traxoline may well be one of our most lukized snezlaus in the future because of our zionter lescelidge.

DIRECTIONS: Answer the following sentences in complete sentences. Be sure to use your best handwriting

1. **What is traxoline?**
2. **Where is traxoline monotilled?**
3. **How is traxoline quaselled?**
4. **Why is it important to know about traxoline?**

Developing Mathematical Thinking Through Questioning

Almost everyone scores 100% on this test. However, no one knows anything about Traxoline, nor do they care.

Even so, if this had been a real quiz, each person would have received an "A."

That is why we must be careful when using multiple-choice questions or questions that are simply rote memorization with no indication of understanding. This is also why it is important to ask students to show their work and why you should give partial credit when the correct process is there.

Studying for a test or quiz can be a game in many ways, and many students have learned to "play" without actually absorbing any knowledge.

Most students can answer these questions. Students and teachers alike have sets of well-developed strategies for producing correct answers to questions that they do not understand.

While this is sometimes a useful skill, we aspire to more in our mathematics classes.

(Seizing Opportunities, AAAS, 1997)

Developing Mathematical Thinking with Effective Questions

To help students build confidence and rely on their own understanding, ask…

- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you make a model to show that?

To help students learn to reason mathematically, ask…

- Is that true for all cases? Explain.
- Can you think of a counterexample?
- How would you prove that?
- What assumptions are you making?

To check student progress, ask…

- Can you explain what you have done so far? What else is there to do?
- Whey did you decide to use this method?
- Can you think of another method that might have worked?
- Is there a more efficient strategy?
- What do you notice when…?
- Why did you decide to organize your results like that?
- Do you think this would work with other numbers?
- Have you thought of all the possibilities? How can you be sure?

To help students collectively make sense of mathematics, ask…

- What do you think about what _____ said?
- Do you agree? Why or why not?
- Does anyone have the same answer but a different way to explain it?
- Do you understand what _____ is saying?
- Can you convince the rest of us that your answer makes sense?

To encourage conjecturing, ask…

- What would happen if…? What if not?
- Do you see a pattern? Can you explain the pattern?
- What are some possibilities here?
- Can you predict the next one? What about the last one?
- What decision do you think he/she should make?

To promote problem solving, ask…

- What do you need to find out?
- What information do you have?
- What strategies are you going to use?
- Will you do it mentally? With pencil and paper? Using a number line?
- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

Developing Mathematical Thinking with Effective Questions, cont.

To help when students get stuck, ask…

- How would you describe the problem in your own words?
- What do you know that is not stated in the problem?
- What facts do you have?
- How did you tackle similar problems?
- Could you try it with simpler numbers? Fewer numbers? Using a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?
- Have you compared your work with anyone else? What did other members of your group try?

To make connections among ideas and applications, ask…

- How does this relate to…?
- What ideas that we have learned before were useful in solving this problem?
- What uses of mathematics did you find in the newspaper last night?
- Can you give me an example of…?

To encourage reflection, ask…

- How did you get your answer?
- Does you answer seem reasonable? Why or why not?
- Can you describe your method to us all? Can you explaining why it works?
- What if you had started with… rather than…?
- What if you could only use…?
- What have you learned or found out today?
- Did you use or learn any new words today? What did they mean? How do you spell them?
- What are the key points or big ideas in this lesson?

The contents of this sheet were developed under a grant from the U.S. Department of Education. However, the contents do not necessarily represent the policy of the U.S. Department of Education, and you should not assume endorsement by the federal government.

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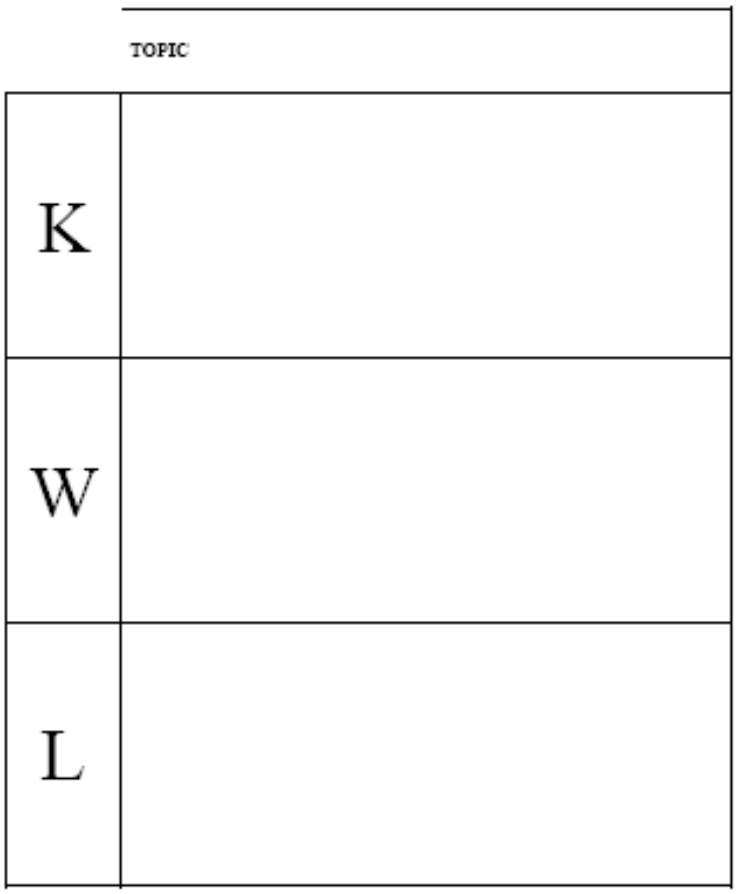
Formative and Summative Assessment: K-W-L Diagram

In the space provided, complete the K and W sections of the K-W-L chart below on the following topic: FORMATIVE & SUMMATIVE ASSESSMENT.



K-What do I know?

W-What do I want to find out?

L-What did I learn?

Formative Assessment Strategies

- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____

Card Sort: Formative vs. Summative Assessment

FORMATIVE ASSESSMENT	SUMMATIVE ASSESSMENT

Card Sort Cards

Teachers, students and parents are the primary users	Used to provide information on what and how to improve achievement
During learning	Used by teachers to identify and respond to student needs
After learning	Used to certify student competence
Teachers, principals, supervisors, program planners, and policy makers are the primary users	Purpose: improve learning
Periodic	Purpose: document achievement of standards
Used to rank and sort students	Primary motivator: threat of punishment, promise of reward
Examples: final exams, placement tests, state assessments, unit tests	Primary motivator: belief that success is achievable
Examples: peer assessment, using rubrics with students, descriptive feedback	Continuous

Table 3-2

Aligning Achievement Targets to Assessment Methods

Target to Be Assessed	*Assessment Method*			
	Selected Response	Extended Written Response	Performance Assessment	Personal Communication
Knowledge Mastery	Good match for assessment mastery of elements of knowledge.	Good match for tapping understanding of relationships among elements of knowledge.	Not a good match—too time consuming to cover everything.	Can ask questions, evaluate answers and infer mastery—but a time-consuming option.
Reasoning Proficiency	Good match only for assessing understanding of some patterns of reasoning.	Written descriptions of complex problem solutions can provide a window into reasoning proficiency.	Can watch students solve some problems and infer reasoning proficiency.	Can ask student to "think aloud" or can ask followup questions to probe reasoning.
Skills	Not a good match. Can assess mastery of the knowledge prerequisites to skillful performance, but cannot rely on these to tap the skill itself.		Good match. Can observe and evaluate skills as they are being performed.	Strong match when skill is oral communication proficiency; not a good match otherwise.
Ability to Create Products	Not a good match. Can assess mastery of the knowledge prerequisites to the ability to create products, but cannot assess the quality of products themselves.	Strong match when the product is written. Not a good match when the product is not written.	Good match. Can assess the attributes of the product itself.	Not a good match.

Source: Adapted from *Student-Involved Assessment FOR Learning*, 4th ed. (p. 69), by R. J. Stiggins, 2005, Upper Saddle River, NJ: Merrill/Prentice Hall. Copyright ©2005 by Pearson Education, Inc. Adapted by permission of Pearson Education, Inc.

Descriptive or Evaluative Feedback

Mark each example of **descriptive feedback** with a ***D*** and each example of **evaluative feedback** with an ***E***. If you believe it is **neither**, mark it with an ***X***.

_____ Good job!

_____ Sloppy work

_____ How did you reach that conclusion? Where's your data?

_____ Proficient

_____ ☺

_____ Your calculations are accurate. Take another look at appropriate units for density.

_____ C-

_____ Excellent!

_____ You need to try harder next time. You can do it!

_____ The students at station two are ready for the lab, they have their books cleared and their safety glasses on.

_____ ✯

_____ You need to label the x-axis, include units with your label, choose an appropriate scale, show the points you plotted, and give the graph a title.

_____ 81%

Self Assessment Task

I did these really well:

1.

2.

I could have:

1.

2.

Next time I need to focus on:

1.

2.

Peer Assessment Task

You did these really well:

1.

2.

You could have:

1.

2.

Next time you need to focus on:

1.

2.

Think/Pair/Share

- How effective are you at giving feedback?
- How do you use formative assessment in your teaching?
- How does formative assessment fit with the new TN Mathematics Standards?
- What could you do differently to make your assessment practices more effective?

Standards Alignment